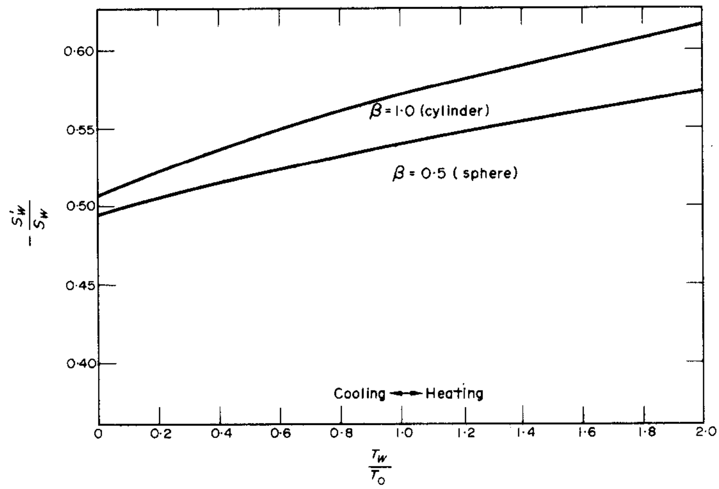

FIG. 3. Variation of S'_w/S_w with the temperature ratio T_w/T_0.

the function F for $\theta = 0°$ (stagnation point) depends nearly linearly on M_∞. F can thus be approximated by the following expression which is also shown in Fig. 2 for the Mach number range $2 < M_\infty < 7$. For

$$\theta = 0°: \quad F = 0{\cdot}48 + 0{\cdot}774\,.\,M_\infty. \tag{9}$$

In the limiting case for $M_\infty \to \infty$ the function F at the stagnation point is given by [6]:

$$F = 0{\cdot}8931\,.\,M_\infty. \tag{10}$$

Using the expression (9) equation (8) can be rewritten for the stagnation point heat transfer for $2 < M_\infty < 7$

$$\frac{Nu_\infty}{(Re_\infty)^{0\cdot5}} = -\frac{S'_w}{S_w}(Pr)^{0\cdot4}\left(\frac{C}{\beta}\right)^{0\cdot5}(0{\cdot}48 + 0{\cdot}774\,.\,M_\infty). \tag{11}$$

The pressure gradient parameter β for the stagnation region is equal to 1·0 for a cylinder and 0·5 for a sphere using Mangler's [5] transformation [4].

The ratio S'_w/S_w is calculated using the analysis of [1] for $\beta = 0{\cdot}5$ and 1·0. The results are shown in Fig. 3 plotted vs the temperature ratio T_w/T_0.

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GAS ABSORPTION INTO A TURBULENT LIQUID FILM

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NOMENCLATURE

a, parameter in equation (1) $[T^{-1}]$;
A_n, constants in equation (12);
B_n, constants in equation (12);
$c_{n,k}$, constants in equation (12);
C, concentration of solute $[ML^{-3}]$;
C_0, inlet concentration $[ML^{-3}]$;
C_s, interfacial concentration $[ML^{-3}]$;
d, film thickness $[L]$;
D, diffusion coefficient $[L^2T^{-1}]$;
h_n, constants defined by equation (18);

k_c, local mass-transfer coefficient [LT^{-1}];
N_A, mass flux [$ML^{-2}T^{-1}$];
Sh, local Sherwood number $= \dfrac{k_c d}{D}$;
v_x, axial velocity [LT^{-1}];
v_0, surface velocity in film [LT^{-1}];
x, axial coordinate [L];
X, dimensionless axial coordinate $= \dfrac{xa}{v_0}$;
X_1, dimensionless axial coordinate $= 4X$;
y, transverse coordinate measured from the interface [L];
Y, dimensionless transverse coordinate $= y\sqrt{\left(\dfrac{a}{d}\right)}$.

Greek symbols

β, dimensionless eddy diffusivity parameter $= \dfrac{ad^2}{D}$;
ε, eddy diffusivity of mass [L^2T^{-1}];
η, pseudo-similarity coordinate $= \dfrac{Y}{2\sqrt{X}}$;
θ, dimensionless concentration $= \dfrac{C_s - C}{C_s - C_0}$;
θ_n, dimensionless functions defined by equations (7) and (8);
ψ, dimensionless group $= Sh\sqrt{\left(\dfrac{\pi X}{\beta}\right)}$.

Mathematical functions

erf, error function, $\operatorname{erf} z = \dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^z e^{-t^2}\,dt$;
i^k erfc, kth repeated integral of the error function:

$$i^k \operatorname{erfc}(z) = \int_z^\infty i^{k-1}\operatorname{erfc}(t)\,dt, \quad (k = 0, 1, 2 \ldots),$$

$$i^{-1}\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} e^{-z^2};$$

δ_{ij}, Kronecker delta function, $\delta_{ij} = 0$, $i \neq j$; $\delta_{ij} = 1$, $i = j$.

LIQUID-PHASE-CONTROLLED mass transfer into a turbulent liquid film is of importance in quite a few process applications as well as in the reaeration of natural streams. Several models have been used for analyzing turbulent transport in the liquid phase. This note is concerned with the eddy diffusivity model which, as Sandall [1] points out in a recent communication, has been used for describing mass transfer in the vicinity of a free surface by quite a few authors (Levich [2], King [3], Davies [4]). The experiments of Lamourelle and Sandall [5] indicate that this eddy diffusivity can be described by

$$\varepsilon = ay^2 \tag{1}$$

where y is the distance measured from the free surface.

For short contact times, the concentration distribution is confined to a region near the free surface, and hence, the usual penetration theory assumptions can be made to simplify the convective diffusion equation. Sandall, in [1], solved this equation using a finite difference technique. He also calculated a two-term approximation of the extended penetration theory solution using numerical means. The purpose of this note is to indicate that this latter solution can be written in terms of elementary functions which makes it possible not only to calculate it to greater accuracy with ease but also permits a better understanding of the nature of the solution itself.

The concentration distribution of solute in the liquid film will satisfy the convective diffusion equation,

$$v_x(y)\frac{\partial C}{\partial x} = \frac{\partial}{\partial y}[D + \varepsilon(y)]\frac{\partial C}{\partial y}. \tag{2}$$

The boundary conditions on $C(x, y)$ are

$$C(0, y) = C_0 \tag{3a}$$

$$C(x, 0) = C_s \tag{3b}$$

$$\frac{\partial C}{\partial y}(x, d) = 0. \tag{3c}$$

Confining our attention to the inlet region and making the usual assumptions of the penetration theory, equations (2) and (3) may be written in dimensionless form as

$$\frac{\partial \theta}{\partial X} = \frac{\partial}{\partial Y}(1 + Y^2)\frac{\partial \theta}{\partial Y} \tag{4}$$

$$\theta(0, Y) = \theta(X, \infty) = 1 \tag{5a}$$

$$\theta(X, 0) = 0. \tag{5b}$$

Upon transforming equation (4) to the (X_1, η) system and writing a solution of the form

$$\theta = \sum_0^\infty \theta_n(\eta) X_1^n \tag{6}$$

the functions $\theta_n(\eta)$ will satisfy

$$\theta_n'' + 2\eta\theta_n' - 4n\theta_n = -(\eta^2\theta_{n-1}'' + 2\eta\theta_{n-1}'), \quad (n = 0, 1, 2 \ldots) \tag{7}$$

with the convention that $\theta_{-1} \equiv 0$. The boundary conditions on $\theta_n(\eta)$ may be written from equations (5) and (6) as

$$\left.\begin{aligned} \theta_n(0) &= 0 \quad (8a) \\ \theta_n(\infty) &= \delta_{n0} \quad (8b) \end{aligned}\right\} (n = 0, 1, 2 \ldots).$$

The solutions of equations (7) and (8) for $n = 0$, 1 and 2 are

$$\theta_0 = \operatorname{erf}\eta \tag{9}$$

$$\theta_1 = \frac{e^{-\eta^2}}{\sqrt{\pi}}\left(\frac{1}{4}\eta - \frac{1}{3}\eta^3\right) \tag{10}$$

$$\theta_2 = \frac{e^{-\eta^2}}{\sqrt{\pi}}\left(-\frac{1}{64}\eta - \frac{11}{96}\eta^3 + \frac{7}{30}\eta^5 - \frac{1}{18}\eta^7\right). \tag{11}$$

In general, the solution of equation (7) may be written as

$$\theta_n = A_n i^{2n}\operatorname{erfc}(\eta) + B_n i^{2n}\operatorname{erfc}(-\eta) + \sum_{k=1}^{2n} c_{n,k}\eta^{2k-1}e^{-\eta^2}. \tag{12}$$

Application of the boundary conditions on $\theta_n(\eta)$ shows that for $n \neq 0$, $A_n = B_n \equiv 0$. The coefficients $c_{n,k}$ are given by the following recursive relations.

$$c_{n,2n} = \frac{1}{3n}c_{n-1,2n-2} \tag{13}$$

$$c_{n,k} = \frac{1}{2(n+k)}[k(2k+1)c_{n,k+1} + k(2k-1)c_{n-1,k} - (4k-3)c_{n-1,k-1} + 2c_{n-1,k-2}],$$
$$(k = 1, 2, 3 \ldots \{2n-1\}), \quad (n = 2, 3, 4 \ldots) \tag{14}$$

with the understanding that $c_{n-1,-1} = c_{n-1,0} = c_{n-1,2n-1} = 0$ in equation (14).

If the mass-transfer coefficient k_c is defined by

$$N_A\Big|_{y=0} = -D\frac{\partial C}{\partial y}\Big|_{y=0} = k_c(C_s - C_0) \tag{15}$$

the Sherwood number Sh for the inlet region may be written as

$$Sh = \beta^{1/2}\left(\frac{2}{\sqrt{(\pi X_1)}} + \sum_{n=1}^\infty c_{n,1}X_1^{n-1/2}\right). \tag{16}$$

Table 1. The coefficients, h_n

n	h_n
1	0·5
2	−0·125
3	0·09583
4	−0·11525
5	0·18469
6	−0·36998

It is convenient to report the results in terms of the dimensionless group, ψ.

$$\psi(X) = Sh\sqrt{\left(\frac{\pi X}{\beta}\right)} = \sum_{n=0}^{\infty} h_n X^n \tag{17}$$

where

$$h_0 = 1 \tag{18a}$$

and

$$h_n = \frac{4^n}{2} c_{n,1}, \quad (n = 1, 2, 3 \ldots). \tag{18b}$$

The coefficients h_n are independent of the parameter β as are the $c_{n,k}$ and hence can be calculated once and for all. The values of h_n for $n = 1$ to 6 are listed in Table 1. Caution should be exercised in using equation (17) since it is an asymptotic expansion, and therefore, inclusion of more terms *will not always* improve the accuracy of the result. In fact, the truncation error would be of the same order of magnitude as the last term included and for best results, summation should be terminated at the point when successive terms start increasing in magnitude. Examination of the coefficients h_n reveals that equation (17) is practically useful only for $X \lesssim 1$.

h_0 corresponds to the classical penetration theory result while h_1 is the equivalent of the additional coefficient calculated by Sandall [1] using numerical methods (he actually calculated $2h_1/\sqrt{\pi}$). Sandall neglected the higher order terms and adjusted h_1 to fit his finite difference results.

Equation (17), when truncated at the Nth term, reads

$$\psi(X) = \sum_{n=0}^{N} h_n X^n \tag{19}$$

Table 2 compares values of ψ calculated for various X values from equation (19) for $N = 1, 2, 4$ and 6 with the corresponding values calculated from the linear fit to the finite difference results proposed by Sandall [1]. One may observe from the table that equation (19) with $N = 2$ would probably be sufficient to bring the extended penetration theory results close enough to the finite difference calculations of Sandall in the region of validity of the former solution.

Table 2. Values of ψ calculated from equation (19) compared with ψ calculated using equation (12) from [1]

	ψ [from equation (19) of this work]				ψ
X	$N = 1$	$N = 2$	$N = 4$	$N = 6$	{from equation (12) of [1]}
0·01	1·005	1·00499	1·00499	1·00499	1·00316
0·05	1·025	1·02469	1·02470	1·02470	1·01717
0·1	1·050	1·04875	1·04883	1·04884	1·03467
0·5	1·250	1·21875	1·22353	1·22352	1·17469
0·8	1·400	1·32000	1·32186	1·28539	1·27971
1·0	1·500	1·37500	1·35558	1·17028	1·34972

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